

Linear Algebra

Q: Find the eigen values of  $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}$

Soln

$$A - \lambda I = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A - \lambda I = \begin{bmatrix} -\lambda & 1 & 2 \\ 1 & -\lambda & -1 \\ 2 & -1 & -\lambda \end{bmatrix}$$

$\therefore$  characteristic eqn is

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} -\lambda & 1 & 2 \\ 1 & -\lambda & -1 \\ 2 & -1 & -\lambda \end{vmatrix} = 0 \Rightarrow -\lambda(\lambda^2 - 1) - (-\lambda + 2) + 2(-1 + 2\lambda) = 0$$

$$\Rightarrow -\lambda^3 + \lambda + \lambda + 2 - 2 + 4\lambda = 0$$

$$\Rightarrow -\lambda^3 + 6\lambda - 0 = 0 \Rightarrow \lambda^3 - 6\lambda + 4 = 0$$

$$\Rightarrow \lambda^3 - 2\lambda^2 + 2\lambda^2 - 4\lambda - 2\lambda + 4 = 0$$

$$\Rightarrow \lambda^2(\lambda - 2) + 2\lambda(\lambda - 2) - 2(\lambda - 2) = 0$$

$$(\lambda - 2)(\lambda^2 + 2\lambda - 2) = 0$$

Either  $\lambda - 2 = 0 \Rightarrow \lambda = 2$  (2)

or  $\lambda^2 + 2\lambda - 2 = 0$

$$\Rightarrow \lambda = \frac{-2 \pm \sqrt{4 + 8}}{2}$$

$$\Rightarrow \lambda = \frac{-2 \pm 2\sqrt{3}}{2} = -1 \pm \sqrt{3}.$$

Thus  
~~Hence~~,  $\lambda = 2, -1 + \sqrt{3}, -1 - \sqrt{3}$ .

Hence, the eigen values of the given matrix are  $2, -1 + \sqrt{3}, -1 - \sqrt{3}$ .

Q. Find the eigen values of

$$A = \begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}.$$

Soln

$$A - \lambda I = \begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A - \lambda I = \begin{bmatrix} a-\lambda & h & g \\ 0 & b-\lambda & 0 \\ 0 & c & c-\lambda \end{bmatrix}$$

(3)

characteristic eqn is

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} a-\lambda & h & g \\ 0 & b-\lambda & 0 \\ 0 & c & c-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (b-\lambda)[(a-\lambda)(c-\lambda) - 0] = 0$$



$$\Rightarrow (a-\lambda)(b-\lambda)(c-\lambda) = 0$$

$$\Rightarrow \lambda = a, b, c.$$

Hence the eigen values of the given matrix are  $a, b, c$ .